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World volume noncommutativity versus target space noncommutativity

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Abstract: It is known that the noncommutativity of D-brane coordinate is responsible for describing the higher-dimensional D-branes in terms of more fundamental ones such as D-particles or D-instantons, while considering a noncommutative torus as a target space is conjectured to be equivalent to introducing the background antisymmetric tensor field in matrix models. In the present paper we clarify the dual nature of both descriptions. Namely the noncommutativity of conjugate momenta of the D-brane coordinates realizes the target space structure, whereas noncommutativity of the coordinates themselves realizes world volume structure. We explicitly construct a boundary state for the Dirichlet boundary condition where the string boundary is adhered to the D-brane on the noncommutative torus. There are non-trivial relations between the parameters appeared in the algebra of the coordinates and that of the momenta.

Keywords: D-branes, M(atrix) Theories, Brane Dynamics in Gauge Theories.

Contents

1. Introduction

Recent developments in nonperturbative string theory have revealed various kinds of noncommutative nature. One of such noncommutativity is that the space-time coordinates of N D-branes are promoted to non-commuting $N \times N$ matrices [1]. In particular, this description of coordinates of D-particles exhibits the noncommutative nature at the sub-string scale [2] where conventional descriptions of space-time by the differential geometry cease to make sense. These facts suggest that noncommutative geometry (NCG) may play an important role as a mathematical tool in the nonperturbative formulation of string theory. The central idea of NCG is, using the equivalence of a manifold and the C^*-algebra of functions defined on it, to reformulate the geometry of manifold in terms of the commutative C^*-algebra and then to generalize the results to the case of a noncommutative algebra. Thus in the spirit of NCG, the world volumes of D-branes should realize NCG because the coordinate functions on the world volumes of D-branes are non-commuting matrices. In fact, the size N of the non-commuting matrices originates not from the target space structure, but from the number of the world volumes of D-branes. Henceforth we will refer to this noncommutative nature associated to the world volumes of D-branes as *world volume noncommutativity* which is characterized by the noncommutativity of the matrices of the embedding coordinates of D-branes. Especially for the classical solutions describing D-brane with dimensions higher than the original one, say D-instanton, we need to set the following non-trivial commutation relations for the D-instanton coordinates

$$[q^i, q^j] = -if^{ij}\,, \tag{1.1}$$

while the commuting case corresponds merely to multiple D-instantons configuration.

Another kind of noncommutative nature in string theory has emerged in the context of the toroidal compactification of Matrix theory [3]. It is well known that Matrix theory compactified on a torus is obtained as supersymmetric Yang-Mills (SYM) theory on the dual torus [4]. However, if we consider the compactification on a torus T^d for $d \geq 2$, we have an additional moduli arising from the 3-form tensor field in the low energy 11-dimensional supergravity. In fact, it was proposed that Matrix theory compactified on a torus with this constant background 3-form field is described by SYM theory on the dual noncommutative torus [5]. Moreover, it is shown that when compactified on T^2 with a constant Neveu-Schwarz 2-form field B_{ij}, D-brane world volume theory is also naturally described by gauge theory on the noncommutative torus and that θ-parameter which characterizes the noncommutativity of the torus should be identified with B_{ij} [6]. Evidences and generalizations of these proposals have been studied in many papers [7]–[26]. In the following, we will call such noncommutative nature characterized by $\theta \sim B_{ij}$ *target space noncommutativity* because it originates from the noncommutativity of the target space torus on which the D-brane world volume is compactified.

Since both noncommutative natures have been revealed as nonperturbative aspects of string theory, it should be important from the point of view of nonperturbative formulation of string theory to analyze them, in particular, to clarify their relationship. This is the problem we address in this paper. We will show that they are in some sense dual to each other and obey a certain simple relation by considering a boundary state which describes a configuration of D-instantons on the (non)commutative two-torus. We will only consider the bosonic string in order to make our argument as simple as possible, but the generalization to the supersymmetric case is straightforward. In our analysis, one can study how moduli of a D-string made of D-instantons are encoded in the noncommutativities in the original D-instanton configuration. From the point of view of D-instantons as the fundamental degrees of freedom like [27, 28, 29, 30], it is important to interpret the various moduli of the higher branes made of D-instantons and symmetries between them as degrees of freedom present in the D-instanton configuration, partly done in [31]. Thus our results will serve as a key ingredient to seek the constructive definition of string theory by regarding D-instantons as the constituent.

The organization of this paper is as follows. In section 2, we present a prescription for the compactification on the noncommutative torus T^2_θ. In section 3, we construct a boundary state corresponding to D-instantons on T^2_θ and show that in this realization the "momentum" operators of the D-instantons have noncommutative nature characterized by $-\theta$. Thus we manifest that the world volume noncommutativity and the target space noncommutativity are dual to each other. In section 4, a configuration of D-instantons whose coordinates are given by non-commuting matrices is studied on T^2 with the constant B_{ij} flux. Following the boundary state analysis done in [31], it is found that the world volume and the target space noncommutativity are related through a simple equation which is nothing but the consistency condition for a twisted bundle on T_θ^2. Implications of this result are also given. Section 5 is devoted to the discussions.

2. Compactification on noncommutative two-torus

In this section, following the prescriptions given in [3, 4, 8], we present a formulation of the compactification on a noncommutative two-torus.

We begin by recalling the usual compactification on T^2:

$$X^i \in \mathbb{R}^2/2\pi\Lambda^2 , \qquad \Lambda^2 \simeq \mathbb{Z}^2 , \qquad p_i \in (\Lambda^2)^*$$
$$\to \exp ip_i(X^i + 2\pi L^i) = \exp ip_i X^i , \qquad \text{for } {}^\forall L^i \in \Lambda^2. \tag{2.1}$$

Generalizing this, we define the compactification on the noncommutative two-torus T_θ^2 as follows. First we introduce the projective representation of $\mathbb{Z}^2$ which is labeled by an element θ of the group $H^2(\mathbb{Z}^2, \mathrm{U}(1)) = \mathrm{U}(1)$ of equivalence classes of 2-cocycles defined up to the coboundary of 1-cocycles:

$$U_1 U_2 = e^{2\pi i\theta} U_2 U_1. \tag{2.2}$$

Then the compactified theory is obtained by restricting to the subspace of the coordinates X's which are invariant under the $\mathbb{Z}_\theta^2$ action:

$$U_i^{-1} X^j U_i = X^j + 2\pi\ell_s \delta_i^j , \qquad i, j = 1, 2 . \tag{2.3}$$

This condition is known as the quotient condition. Thus (2.3) means that

$$X^i \in \mathbb{R}^2/\mathbb{Z}_\theta^2 ,$$

where $\mathbb{Z}_\theta^2$ denotes the projective representation (2.2). Note that (2.2) happens to define the algebra of functions $\mathcal{A}(T_\theta^2)$ on a noncommutative torus T_θ^2.

In [18], a concrete representation for each θ is given by introducing a quantum plane coordinate

$$[\sigma_i, \sigma_j] = 2\pi i\theta\epsilon_{ij} . \tag{2.4}$$

Then operators defined as

$$\tilde{U}_i = e^{i\sigma_i} , \tag{2.5}$$

generate the algebra $\mathcal{A}(T^2_{-\theta})$ which is (one of) the dual algebra to $\mathcal{A}(T_\theta^2)$ (see below):

$$\tilde{U}_1 \tilde{U}_2 = e^{-2\pi i\theta} \tilde{U}_2 \tilde{U}_1 . \tag{2.6}$$

Using σ_i and their partial derivative operators satisfying

$$[\partial_i, \sigma_j] = \delta_{ij} , \qquad [\partial_i, \partial_j] = 0 ,$$

U_i and X^i can be expressed as

$$
\begin{aligned}
U_i &= e^{i\sigma_i} e^{2\pi\theta\epsilon_{ij}\partial_j} \,, \\
X^i &= q^i + A^i(\tilde{U}_i) \,, \\
q^i &= -2\pi i \ell_s D_i, \qquad D_1 = \partial_1 \,, \qquad D_2 = \partial_2 - if\sigma_1 \,,
\end{aligned}
\tag{2.7}
$$

where A^i is an arbitrary function of $\tilde{U}_i$. For later convenience, we have taken a representation in such a way that q^i's are noncommutative:

$$
[q^1, q^2] = -ik \,, \qquad k \equiv -(2\pi\ell_s)^2 f \,. \tag{2.8}
$$

In D-brane matrix models q^i correspond to a classical solution to the equation of motion for X^i and A^i correspond to a fluctuation around q^i.

Then the 'dual lattice' corresponding to T_θ^2 is given as the algebra of all operators that commute with U_i [5]. In [14, 18], this algebra is represented as that of the sections on the twisted bundle of the adjoint representation of $\mathrm{U}(n)$ on T_θ^2 in the following way. First, introduce $n \times n$ matrices U, V satisfying

$$
UV = e^{-2\pi i m/n} VU \,, \tag{2.9}
$$

where m is an integer. Without loss of generality, we assume that n and m are relatively prime. Taking an appropriate basis, such matrices are represented as

$$
U_{kl} = \exp(2\pi i k m/n)\delta_{kl} \,, \qquad V_{kl} = \delta_{k+1,l} \,. \tag{2.10}
$$

Then the sections on adjoint bundles are generated by operator-valued matrices expressed as

$$
Z_1 = \exp\left(i\frac{\sigma_1}{n - m\theta}\right) V^b \,, \qquad Z_2 = \exp\left(i\frac{\sigma_2}{n}\right) U^{-b} \,, \tag{2.11}
$$

where b is an integer satisfying $an - bm = 1$ for some integer a. They indeed commute with the U_i's and are also generators of the algebra of functions on a new noncommutative torus $\mathbb{Z}_{\theta'}^2$ where

$$
\theta' = \frac{a(-\theta) + b}{m(-\theta) + n} \,. \tag{2.12}
$$

Under this representation, the momentum lattice corresponding to (2.3) is nothing other than $\mathbb{Z}_{\theta'}^2$. In fact, for any element $Z_i \in \mathbb{Z}_{\theta'}^2$, an operator z_i defined by $e^{iz_i} = Z_i$ satisfies

$$
\exp i\mathrm{Tr}(U_i^{-1} X^j U_i z_j) = \exp i\mathrm{Tr}(U_i^{-1} X^j z_j U_i) = \exp i\mathrm{Tr}(X^j z_j) \,, \tag{2.13}
$$

in the analogy of (2.1). Here Tr denotes the composition of the trace tr over $n \times n$ matrices and the integration over σ_i [14]:

$$
\mathrm{Tr}(f(Z_1, Z_2)) = \int d\sigma_1 d\sigma_2 \mathrm{tr} f(Z_1, Z_2) \,. \tag{2.14}
$$

As shown in [4, 14, 18], in the context of Matrix theory unitary operators U_i and scalar fields X^i are represented as operators (2.7) for $k \neq 0$ acting on the twisted $U(n)$ fundamental bundle on the (non)commutative torus,

$$\begin{aligned}\phi(\sigma_1 + 2\pi, \sigma_2) &= \Omega_1(\sigma_1, \sigma_2)\phi(\sigma_1, \sigma_2)\,,\\ \phi(\sigma_1, \sigma_2 + 2\pi) &= \Omega_2(\sigma_1, \sigma_2)\phi(\sigma_1, \sigma_2)\,,\end{aligned} \tag{2.15}$$

where Ω_i are transition functions

$$\Omega_1(\sigma_1, \sigma_2) = e^{i\alpha\sigma_2}U\,, \qquad \Omega_2(\sigma_1, \sigma_2) = V\,, \tag{2.16}$$

with U, V given in (2.10) and α being a certain number determined below. They satisfy the consistency condition

$$\Omega_1(\sigma_1, \sigma_2 + 2\pi)\Omega_2(\sigma_1, \sigma_2) = \Omega_2(\sigma_1 + 2\pi, \sigma_2)\Omega_1(\sigma_1, \sigma_2)\,. \tag{2.17}$$

On the other hand, covariant derivatives which appear in (2.7) satisfy the twisted boundary conditions

$$\begin{aligned}D_i(\sigma_1 + 2\pi, \sigma_2) &= \Omega_1(\sigma_1, \sigma_2)D_i(\sigma_1, \sigma_2)\Omega_1(\sigma_1, \sigma_2)^{-1}\,,\\ D_i(\sigma_1, \sigma_2 + 2\pi) &= \Omega_2(\sigma_1, \sigma_2)D_i(\sigma_1, \sigma_2)\Omega_2(\sigma_1, \sigma_2)^{-1}\,.\end{aligned} \tag{2.18}$$

Combined with (2.17), this leads to the remarkable relation

$$\frac{1}{2\pi f} + \theta = \frac{n}{m}\,. \tag{2.19}$$

This equation means that the magnetic flux, the noncommutativity of the base space torus, the rank of the gauge group and the number of the twist must be related with each other from the requirement of the gauge theory on the noncommutative torus, since $[D_1, D_2] = -if$ and $\alpha = m/n$.

3. D-instantons on T^2 and T^2_θ

In this section, we consider a boundary state together with operators acting on it corresponding to a configuration of D-instantons on T^2 and T^2_θ in the bosonic string theory. Then one can show that the target space noncommutativity is reflected by that of the momentum operators and hence noncommutative structures in the D-instanton world volume and the target space are dual to each other. In order to clarify the role of the noncommutativities, we proceed step by step starting from the commutative case ($\theta = 0, k = 0$).

The case $\theta = 0$, $k = 0$. Let us begin with defining a boundary state corresponding to D-instantons on commutative torus T^2. We propose that it takes the following form:

$$|\sigma) = \sum_{w^i \in \mathbb{Z}} e^{i\sigma_i w^i} |X^i = 2\pi\ell_s w^i\rangle_{-1}\,, \qquad \sigma_i \in \mathbb{R}/2\pi\mathbb{Z}\,, \qquad i = 1, 2\,, \tag{3.1}$$

where $|X^i = 2\pi\ell_s w^i\rangle_{-1}$ denotes the Dirichlet boundary state defined as

$$|X^i = 2\pi\ell_s w^i\rangle_{-1} = e^{-2\pi i\ell_s p_i w^i} |X^i = 0\rangle_{-1}\,. \tag{3.2}$$

Here we define $|X^i = 0\rangle_{-1}$ to be the coherent state satisfying

$$X^i(s)|X^i = 0\rangle_{-1} = 0\,, \tag{3.3}$$

and p_i is the center of mass momentum of the string. Since (3.2) labeled by $w_i \in \mathbb{Z}$ are all physically equivalent in the torus compactification, it is natural to consider the state $|\sigma)$ given by a superposition of these states as in (3.1). Then it is easy to show that on $|\sigma)$ the center of mass coordinate of the string x^i (conjugate to p_i) becomes

$$x^i|\sigma) = -2\pi i\ell_s \partial_i |\sigma)\,, \tag{3.4}$$

and, therefore, the coordinate operator of D-instantons q^i is represented as

$$q^i = -2\pi i\ell_s \partial_i\,. \tag{3.5}$$

On the other hand, on $|\sigma)$, p_j satisfies[1]

$$\left(p_j - \frac{\sigma_j}{2\pi\ell_s}\right)|\sigma) = 0\,, \tag{3.6}$$

since by definition,

$$e^{2\pi i\ell_s p_j}|X^i = 2\pi i\ell_s w^i\rangle_{-1} = |X^i = 2\pi\ell_s(w^i - \delta^j_i)\rangle_{-1}\,, \tag{3.7}$$

and hence

$$\begin{aligned} e^{2\pi i\ell_s p_j}|\sigma) &= \sum_{w^i} e^{i\sigma_i w^i}|X^i = 2\pi\ell_s(w^i - \delta^j_i)\rangle_{-1} \\ &= \sum_{w^i} e^{i\sigma_i(w^i + \delta^j_i)}|X^i = 2\pi\ell_s w^i\rangle_{-1} \\ &= e^{i\sigma_j}|\sigma) = \tilde{U}_j|\sigma). \end{aligned} \tag{3.8}$$

[1]Since boundary state is a interaction vertex between string and D-brane, (3.4) and (3.6) are analogues of the relations $x_1\delta(x_1 - x_2) = x_2\delta(x_1 - x_2)$ and $-i(\partial_1 + \partial_2)\delta(x_1 - x_2) = 0$ satisfied by the local vertex $\delta(x_1 - x_2)$ for the point particles.

From (3.6), we can identify the 'momentum' π_j carried by the D-instanton configuration as

$$\pi_j = -\frac{\sigma_j}{2\pi\ell_s}, \tag{3.9}$$

because (3.6) represents the total momentum conservation. Note that they indeed satisfy the Heisenberg algebra $[q^i, \pi_j] = i\delta^i_j$.

Comparing (3.5) and (3.9) with (2.5) and (2.7), we see that $|\sigma)$ naturally realizes the representation corresponding to D-instantons on T^2 with $f = 0$.

From (3.1) σ is a dual coordinate to the winding number. This is consistent with usual arguments in Matrix theory where matrices have extra indices corresponding to the multiple cover of S^1 and the following quotient relation is put:

$$X^i_{m,n} = X^i_{m-1,n-1} + 2\pi R I \delta_{mn}\,. \tag{3.10}$$

Then the Fourier transformation with respect to the extra indices leads to dual coordinate σ which gives the base space of the SYM description.

The case $\theta \neq 0$, $k = 0$. Now let us turn to the string theory around D-instantons on the noncommutative torus, namely, $\theta \neq 0$. In this case, it is natural to define a state $|\sigma)$ of the form (3.1) with σ given by (2.4). Note that $|\sigma)$ defined as such is not the eigenstate of the abstract operator σ. It is easy to find that on $|\sigma)$ x^i is again given as

$$x^i|\sigma) = q^i|\sigma)\,, \tag{3.11}$$

with q^i given as (3.5), however, (3.6) is modified as

$$\left(p_j - \frac{\theta}{4\pi\alpha'}\epsilon_{jk}x^k - \frac{\sigma_j}{2\pi\ell_s}\right)|\sigma) = 0\,. \tag{3.12}$$

In order to identify π_j, we propose that it should satisfy Heisenberg algebra with q^i being (3.5) and commute with U_i according to the formulation developed in section 2. Then we find that π_j is again expressed as (3.9). From (3.12), we obtain the total momentum conservation in the following form:

$$\left(p_j - \frac{\theta}{4\pi\alpha'}\epsilon_{jk}x^k + \pi_j\right)|\sigma) = 0\,. \tag{3.13}$$

Note that the center of mass momentum of the string seems to become $p_j - \frac{\theta}{4\pi\alpha'}\epsilon_{jk}x^k$ in (3.13). This form is an analogue of the momentum of a charged particle moving in a magnetic field. In the next section, by considering a configuration of D-instantons on T^2 with a constant 2-form field flux B_{ij}, we will find it quite natural to make the identification $\theta = B_{12}$. Moreover, using (3.11), this equation can be rewritten as

$$\left(p_j - \frac{\theta}{4\pi\alpha'}\epsilon_{jk}q^k + \pi_j\right)|\sigma) = 0\,. \tag{3.14}$$

It states that in the case of $\theta \neq 0$ the momentum operator π_j has an extent introduced by the second term in (3.14) in the momentum space due to the target space noncommutativity and that as a consequence the center of mass momentum of the string p_i satisfies the total momentum conservation in a modified form as above. It would be interesting to examine relation between this extension in the momentum space and the target space noncommutativity. In any case, the representation on $|\sigma)$ with σ given by (2.4) turns out to correspond to a configuration of D-instantons on T^2_θ, since q^i is represented as (2.7) and π_j is on the dual lattice. Remarkably, on $|\sigma)$, the 'momentum operators' of D-instantons do not commute with each other reflecting the noncommutativity of the target space torus:

$$[\pi_1, \pi_2] = \frac{1}{(2\pi\ell_s)^2}[\sigma_1, \sigma_2] = \frac{i}{2\pi\ell_s^2}\theta\,. \tag{3.15}$$

Moreover, for any operator W which depends on σ_i and ∂_i, one can define corresponding matrix elements $W_{m^i n^j}$ as

$$W|\sigma) = \sum_{n^j} W e^{i\sigma_j n^j}|X^j = 2\pi\ell_s n^j\rangle_{-1} = \sum_{m^i, n^j} e^{i\sigma_i m^i} W_{m^i n^j}|X^j = 2\pi\ell_s n^j\rangle_{-1}\,. \tag{3.16}$$

Then if one imposes the quotient condition (2.3) on $X^i = q^i + A^i(\tilde{U}_i)$ with q^i being the coordinate operator of D-instantons given in (3.5), it can be rewritten as the conditions on matrix elements like

$$\begin{aligned} q^j_{n^l+\delta_{il}\ m^k+\delta_{ik}} &= q^j_{n^l m^k} + 2\pi\ell_s\delta_{ij}\delta_{n^l m^k}\,, \\ A^j_{n^l+\delta_{il}\ m^k+\delta_{ik}} &= e^{-\pi i\theta\epsilon_{ik}(m^k-n^k)} A^j_{n^l m^k}\,. \end{aligned} \tag{3.17}$$

Since q^i is a diagonal matrix in this representation, these lead to

$$X^j_{n^l+\delta_{il}\ m^k+\delta_{ik}} = e^{-\pi i\theta\epsilon_{ik}(m^k-n^k)}(X^j_{n^l m^k} + 2\pi\ell_s\delta_{ij}\delta_{n^l m^k})\,. \tag{3.18}$$

Under the identification $\theta = B_{12}$ which we made above, this is nothing other than the relation proposed in [12]. This confirms that in the present case it is natural to choose the representation on $|\sigma)$. Thus we conclude that on T^2_θ D-instanton momentum operators have the noncommutative structure as in (3.15), and this is also the case on T^2 with the constant 2-form field flux.

As emphasized in the introduction, the world volume noncommutativity of D-instantons is characterized by that of the space-time coordinates of these D-instantons, while as shown in this section, the target space noncommutativity causes that of the momentum operators of D-instantons. In this sense we can say that they are dual to each other.

For later convenience, it is useful to see that for $\theta \neq 0$, due to the ordering ambiguity, it is possible that $|\sigma)$ takes a slightly different form from (3.1)

$$|\sigma) = \sum_{w^i\in\mathbb{Z}} e^{i\sigma_2 w^2} e^{i\sigma_1 w^1}|X^i = 2\pi\ell_s w^i\rangle_{-1}\,, \tag{3.19}$$

with σ given by (2.4). q^i is expressed as (3.5) on this state, while (3.13) turns into

$$\left(p_1 - \frac{\theta}{2\pi\alpha'}x^2 + \pi_1\right)|\sigma) = 0\,, \qquad (p_2 + \pi_2)|\sigma) = 0, \tag{3.20}$$

where π_i is the same as (3.9). The modified form (3.19) is formally obtained from (3.1) by multiplying a factor $\exp(i\frac{\theta}{4\pi\alpha'}x^1x^2)$, which may be interpreted as a gauge transformation for the 2-form gauge potential under the identification of $\theta = B_{12}$.

The case $\theta = 0$, $k \neq 0$. Next let us discuss the case of $k \neq 0$. We first consider such a configuration on the commutative torus T^2. We propose that a boundary state corresponding to it is given as

$$|\sigma_1, \sigma_2) = \sum_{w^i \in \mathbb{Z}} e^{i\sigma_i w^i} |2\pi\ell_s(w^1, w^2 - f\sigma_1)\rangle\,, \tag{3.21}$$

where $f = -k/(2\pi\ell_s)^2$, and

$$|2\pi\ell_s(w^1, w^2 - f\sigma_1)\rangle \equiv |(X^1, X^2) = 2\pi\ell_s(w^1, w^2 - f\sigma_1)\rangle\,. \tag{3.22}$$

In fact, it is easy to find that

$$\begin{aligned}(x^1 + (2\pi\ell_s)^2 f p_2)|\sigma_1, \sigma_2) &= -2\pi i\ell_s \partial_{\sigma_1}|\sigma_1, \sigma_2)\,,\\ x^2|\sigma_1, \sigma_2) &= -2\pi i\ell_s(\partial_{\sigma_2} - if\sigma_1)|\sigma_1, \sigma_2)\,,\end{aligned} \tag{3.23}$$

from which we read the coordinates of D-instantons q^i as

$$q^1 = -2\pi i\ell_s \partial_{\sigma_1}\,, \qquad q^2 = -2\pi i\ell_s(\partial_{\sigma_2} - if\sigma_1)\,. \tag{3.24}$$

These imply that $|\sigma_1, \sigma_2)$ exactly provides the representation given in (2.7). Note that in (3.23) x^1 does not quite coincide with q^1. We regard it as a signal that for $k \neq 0$ each D-instanton is spreading in size $\sim k$ and that strings are attached to such an extended object.[2] In order to put a more physical interpretation on what happens, we analyze this configuration by means of another method in the next section. Also note that

$$e^{2\pi i\ell_s p_i}|\sigma_1, \sigma_2) = e^{i\sigma_i}|\sigma_i, \sigma_2)\,, \tag{3.25}$$

and, therefore, we obtain

$$\pi_i = -\frac{\sigma_i}{2\pi\ell_s}\,. \tag{3.26}$$

[2] It may also suggest that for $k \neq 0$ a combination $x^1 - kp_2$ plays a role of a new string coordinate in which the center of mass coordinates of the string itself become noncommutative. A similar result is also obtained in [13, 25] by considering the quantization of open strings ending on D-branes. The noncommutativity of string coordinates already appeared in earlier works [32, 33].

This is again consistent with the formulation in section 2. Moreover, one can generalize (3.21) to include a index $r = 1, \ldots, n$ corresponding to the Chan-Paton index in the open string sector in the following way:

$$|\sigma_1, \sigma_2)^r = \sum_{w^i \in \mathbb{Z}} e^{i\sigma_1 w^1 + i(\sigma_2 + 2\pi r)w^2/n} |2\pi \ell_s (w^1, w^2/n - f\sigma_1)\rangle \,, \tag{3.27}$$

which coincides with (3.21) for $n = 1$. Like (3.21), $|\sigma_1, \sigma_2)^r$ satisfies

$$\begin{aligned} (x^1 + (2\pi\ell_s)^2 f p_2)|\sigma_1, \sigma_2)^r &= -2\pi i \ell_s \partial_{\sigma_1} |\sigma_1, \sigma_2)^r \,, \\ x^2 |\sigma_1, \sigma_2)^r &= -2\pi i \ell_s (\partial_{\sigma_2} - i f \sigma_1)|\sigma_1, \sigma_2)^r \,, \\ e^{2\pi i \ell_s p_i} |\sigma_1, \sigma_2)^r &= e^{i\sigma_i} |\sigma_i, \sigma_2)^r \,. \end{aligned} \tag{3.28}$$

Thus on $|\sigma_1, \sigma_2)^r$ the coordinates of D-instantons q^i are represented as $n \times n$ matrices

$$q^1 = -2\pi i \ell_s \partial_{\sigma_1} I \,, \qquad q^2 = -2\pi i \ell_s (\partial_{\sigma_2} - i f \sigma_1) I \,, \tag{3.29}$$

and π_i are represented as

$$\pi_i = -\frac{\sigma_i}{2\pi \ell_s} I \,, \tag{3.30}$$

as expected. Recalling that as commented in section 2, representations like (3.29) are given on the twisted U(n) fundamental bundle on T^2 in the context of Matrix theory, we expect that the state (3.27) in string theory has structure similar to the twisted U(n) fundamental bundle. In fact, (3.27) satisfies the same twisted boundary condition as in (2.15),

$$\begin{aligned} |\sigma_1 + 2\pi, \sigma_2)^r &= e^{im/n\sigma_2} U_{rs} |\sigma_1, \sigma_2)^s \,, \\ |\sigma_1, \sigma_2 + 2\pi)^r &= V_{rs} |\sigma_1, \sigma_2)^s \,, \end{aligned} \tag{3.31}$$

provided that

$$\frac{1}{2\pi f} = \frac{n}{m} \,, \tag{3.32}$$

which is just the equation obtained by setting $\theta = 0$ in (2.19). Later we will discuss this point including the case of $\theta \neq 0$.

The case $\theta \neq 0$, $k \neq 0$. Finally let us discuss the case of $\theta \neq 0$, $k \neq 0$. Using the quantum plane coordinate (2.4), a boundary state corresponding to a configuration with $\theta \neq 0, k \neq 0$ including the index $r = 1, \ldots, n$ is given as

$$|\sigma_1, \sigma_2)^r = \sum_{w^i \in \mathbb{Z}} e^{i(\sigma_2 + 2\pi r)w^2/n} e^{i\sigma_1 w^1} |2\pi \ell_s (w^1, (1 + 2\pi f \theta) w^2/n - f\sigma_1)\rangle \,. \tag{3.33}$$

Note that this state reproduces (3.27) for $\theta = 0$. Little examination shows that this state satisfies the following equations:

$$(x^1 + (2\pi\ell_s)^2 f p_2)|\sigma_1, \sigma_2)^r = -2\pi i \ell_s \partial_{\sigma_1} |\sigma_1, \sigma_2)^r \,, \tag{3.34}$$

$$x^2 |\sigma_1, \sigma_2)^r = -2\pi i \ell_s (\partial_{\sigma_2} - i f \sigma_1)|\sigma_1, \sigma_2)^r \,, \tag{3.35}$$

$$e^{2\pi i \ell_s \left(p_1 - \frac{\theta}{2\pi\alpha'} x^2\right)} |\sigma_1, \sigma_2)^r = e^{i(1 + 2\pi f \theta)\sigma_1} |\sigma_1, \sigma_2)^r \,, \tag{3.36}$$

$$e^{i(1 + 2\pi f \theta) 2\pi \ell_s p_2} |\sigma_1, \sigma_2)^r = e^{i\sigma_2} |\sigma_1, \sigma_2)^r \,. \tag{3.37}$$

From (3.34) and (3.35), on $|\sigma_1, \sigma_2)^r$ the coordinates of D-instantons are given as

$$q^1 = -2\pi i\ell_s \partial_{\sigma_1} I \,, \tag{3.38}$$
$$q^2 = -2\pi i\ell_s (\partial_{\sigma_2} - if\sigma_1) I \,. \tag{3.39}$$

Next let us identify the momenta π_i carried by the D-instanton configuration. As for π_2, combining (3.37) with the total momentum conservation

$$(p_2 + \pi_2)|\sigma_1, \sigma_2)^r = 0 \,, \tag{3.40}$$

we obtain

$$\pi_2 = -\frac{\sigma_2}{(1 + 2\pi f\theta)2\pi\ell_s} I \,. \tag{3.41}$$

In fact, we see that (3.41) is consistent with both Heisenberg algebra and formulation in section 2 because q^2 is represented as in (3.39), and (3.41) commutes with U_i. As for π_1, however, if we require that π_1 should satisfy the Heisenberg algebra with q^1 given by (3.38) and commute with U_i, it must be given as

$$\pi_1 = -\frac{\sigma_1}{2\pi\ell_s} I \,. \tag{3.42}$$

Compared with (3.36), this equation implies that the total momentum conservation in the 1-direction is modified in the following form:

$$\left(p_1 - \frac{\theta}{2\pi\alpha'} x^2 + (1 + 2\pi f\theta)\pi_1\right)|\sigma_1, \sigma_2)^r = 0 \,, \tag{3.43}$$

or

$$\left(p_1 - \frac{\theta}{2\pi\alpha'} q^2 + (1 + 2\pi f\theta)\pi_1\right)|\sigma_1, \sigma_2)^r = 0 \,. \tag{3.44}$$

These equations admit of the same interpretation as in (3.13) and (3.14). Note that (3.43) agrees with the first equation in (3.20) for $f = 0$. Thus we have verified that $|\sigma_1, \sigma_2)^r$ indeed corresponds to the configuration in the case of $\theta \neq 0$, $k \neq 0$.

As emphasized in (3.31), whether $\theta = 0$ or not, our state $|\sigma_1, \sigma_2)^r$ in the case of $k \neq 0$ satisfies the same twisted boundary condition as in (2.15),

$$\begin{aligned} |\sigma_1 + 2\pi, \sigma_2)^r &= e^{im/n\sigma_2} U_{rs} |\sigma_1, \sigma_2)^s \,, \\ |\sigma_1, \sigma_2 + 2\pi)^r &= V_{rs} |\sigma_1, \sigma_2)^s \,, \end{aligned} \tag{3.45}$$

provided that

$$\frac{1}{2\pi f} + \theta = \frac{n}{m} \,, \tag{3.46}$$

which is just the equation obtained in (2.19). A similar equation is also obtained in [5]. It is worth noticing that a boundary state which corresponds to the twisted fundamental bundle on the (non)commutative torus can be constructed in the context of string theory, and that it reproduces the same condition as that required from a consistency of the twisted bundle. Although at present this correspondence is formal, it is expected that string theory around a D-instanton configuration with $\theta \neq 0$, $k \neq 0$ has certain structure described by the (fundamental, or possibly adjoint) twisted bundle on the noncommutative torus. For example, if (3.46) is satisfied, it is easy to show that

$$e^{2\pi i \ell_s \left(p_1 - \frac{\theta}{2\pi\alpha'} x^2\right)} |\sigma_1, \sigma_2)^r = (Z_1^n)_{rs} |\sigma_1, \sigma_2)^s \,, \tag{3.47}$$

$$e^{i \frac{2\pi \ell_s}{n - m\theta} p_2} |\sigma_1, \sigma_2)^r = (Z_2)_{rs} |\sigma_1, \sigma_2)^s \,. \tag{3.48}$$

We see that operators Z_i defined in (2.11) naturally appear in the right-hand sides of both equations. It may suggest that the statement mentioned above is indeed the case. Among others, it would be important to give a physical meaning to (3.46) in the context of string theory. For this purpose, in the next section we consider a boundary state corresponding to a configuration of D-instantons on T^2 with a constant B_{ij} flux and show that the relation (3.46) is automatically satisfied with suitable identifications.

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4. D-instantons on T^2 with the 2-form field flux

In this section, we consider a configuration of D-instantons on the commutative torus T^2 of size $2\pi\ell_s$ with the constant 2-form field B_{ij} flux. According to the boundary state analysis made in [31], a configuration of D-instantons whose space-time coordinates are noncommutative can be regarded as that of a single D-string. We closely follow this approach and clarify the relationship between the noncommutativity of the world volume and the B_{ij} flux.

A configuration of D-instantons on T^2 we would like to consider is given by

$$[q^1, q^2] = -ik \neq 0 \,, \tag{4.1}$$

where q^i are $n \times n$ matrices corresponding to the space-time coordinates of D-instantons, entries of which are in general noncommutative operators. In compactified Matrix theory, such matrices are explicitly constructed for finite n [4]. Note that the representation (2.7) of q^i given in section 2 shows that k is in proportion to the magnetic flux f as

$$k = -(2\pi\ell_s)^2 f \,, \qquad [D_1, D_2] = -if \,. \tag{4.2}$$

As stated in the introduction, k characterizes the noncommutative structure of the world volume of D-instantons.

The boundary state for the configuration (4.1) is

$$|B\rangle_{-1} = \mathrm{tr} P \exp\left(-i \int_0^{2\pi} ds P_i(s) q^i\right) |X^i = 0\rangle_{-1} \,, \tag{4.3}$$

where $P_i(s)$ is the canonical momentum of the string in the presence of the background 2-form

$$P_i(s) = \frac{1}{2\pi\alpha'}\left(G_{ij}\dot{X}^j - B_{ij}X'^j\right) . \tag{4.4}$$

Then similarly to [31], this boundary state can be expressed as the path integral representation

$$|B\rangle_{-1} = \int [dq^1 dq^2] \exp\left(\frac{i}{k}\int_0^{2\pi} ds q^1(s)\partial_s q^2(s) - i\int_0^{2\pi} ds P_i(s) q^i(s)\right) |X^i = 0\rangle_{-1} . \tag{4.5}$$

From trivial identities

$$0 = \int [dq^1 dq^2] \frac{\delta}{\delta q^j(s)} \exp\left(\frac{i}{k}\int_0^{2\pi} ds q^1(s)\partial_s q^2(s) - i\int_0^{2\pi} ds P_i(s) q^i(s)\right) |X^i = 0\rangle_{-1} , \tag{4.6}$$

one obtains

$$\left[P_i(s) - \frac{1}{k}\epsilon_{ij}\partial_s X^j\right] |B\rangle_{-1} = 0 , \tag{4.7}$$

and, therefore,

$$\begin{aligned} 0 = &\left[\frac{1}{2\pi\ell_s}\left(\hat{n}_i - \frac{2\pi\alpha'}{k}\epsilon_{ij}\hat{w}^j\right) + \right. \\ &\left. + \frac{1}{2\pi}\sqrt{\frac{1}{2\alpha'}}\sum_{n\neq 0}\left(\left(G_{ij} + B_{ij} + \frac{2\pi\alpha'}{k}\epsilon_{ij}\right)\alpha_n^j + \left(G_{ij} - B_{ij} - \frac{2\pi\alpha'}{k}\epsilon_{ij}\right)\tilde{\alpha}_n^j\right) e^{ins}\right] |B\rangle_{-1} , \end{aligned} \tag{4.8}$$

where G_{ij} is the metric of the T^2 and $\hat{n}_i$, $\hat{w}^j$ are momentum number and winding number operator, respectively. On the other hand, the boundary state $|B\rangle_1$ corresponding to a configuration of a D-string on T^2 of the metric G'_{ij} with the background B'_{ij}, F'_{ij} satisfies the following equation:

$$\begin{aligned} 0 &= \left[P_i(s) - F'_{ij}\partial_s X^j(s)\right] |B\rangle_1 \\ &= \left[\frac{1}{2\pi\ell_s}\left(\hat{n}'_i - 2\pi\alpha' F'_{ij}\hat{w}'^j\right) + \right. \\ &\quad \left. + \frac{1}{2\pi}\sqrt{\frac{1}{2\alpha'}}\sum_{n\neq 0}\left((G'_{ij} + B'_{ij} + 2\pi\alpha' F'_{ij})\alpha_n^j + (G'_{ij} - B'_{ij} - 2\pi\alpha' F'_{ij})\tilde{\alpha}_n^j\right) e^{ins}\right] |B\rangle_1 . \end{aligned} \tag{4.9}$$

Comparing (4.8) with (4.9) shows that the configuration of n D-instantons corresponding to (4.1) on T^2 with the B_{ij} background coincides with that of a D-string on T^2 with the background

$$\hat{n}'_i - 2\pi\alpha' F'_{ij}\hat{w}'^j = \hat{n}_i - \frac{2\pi\alpha'}{k}\epsilon_{ij}\hat{w}^j , \tag{4.10}$$

$$G'_{ij} + B'_{ij} + 2\pi\alpha' F'_{ij} = G_{ij} + B_{ij} + \frac{2\pi\alpha'}{k}\epsilon_{ij} , \tag{4.11}$$

$$G'_{ij} - B'_{ij} - 2\pi\alpha' F'_{ij} = G_{ij} - B_{ij} - \frac{2\pi\alpha'}{k}\epsilon_{ij} . \tag{4.12}$$

From the second and third equation, we obtain

$$G_{ij} = G'_{ij}\,, \qquad B_{ij} + \frac{2\pi\alpha'}{k}\epsilon_{ij} = B'_{ij} + 2\pi\alpha' F'_{ij}\,, \tag{4.13}$$

and then 'natural identification' is

$$B'_{ij} = B_{ij}\,, \qquad F'_{ij} = \frac{1}{k}\epsilon_{ij}\,. \tag{4.14}$$

The last equation is exactly the same as in [31]. However, recalling that B'_{ij} and F'_{ij} are transformed to each other by the gauge transformations from the point of view of the D-string, we are tempted to consider that there exist some 'gauge transformations' between $1/k$ and B_{ij} and interpret the identification (4.14) as a particular gauge fixing.

Next let us address the problem of deriving the equation like (2.19) in the D-instantons configuration under consideration. If the U(n) gauge theory on the non-commutative tours correctly describes the n D-instantons on the torus with the B_{ij} flux as proposed in [6, 14, 18], the consistency condition (2.19) must be automatically satisfied in the D-instanton dynamics. As we have shown above, we can regard this configuration as the D-string configuration on T^2 with the background $2\pi\alpha'\mathcal{F}'_{ij} \equiv B'_{ij} + 2\pi\alpha' F'_{ij}$. Then by construction, the D-string have the D-instanton charge n and hence

$$\frac{1}{2\pi}\int_{\text{D-string world volume}} \mathcal{F}' = n\,, \qquad \mathcal{F}' \equiv \mathcal{F}'_{12}\,, \tag{4.15}$$

because $\mathcal{F}'$ couples the Ramond-Ramond scalar field $C^{(0)}$ through the Chern-Simons coupling $\int C \wedge e^{2\pi\alpha'\mathcal{F}'}$. Assuming that the D-string winds m times around T^2,

$$\frac{1}{2\pi}\int_{T^2} \mathcal{F}' = \frac{n}{m}\,. \tag{4.16}$$

According to the second equation in (4.13), this can be reinterpret in terms of the background of the configuration of D-instantons as

$$\frac{2\pi\alpha'}{k} + B_{12} = \frac{n}{m}\,. \tag{4.17}$$

Making the identification $B_{12} = \theta$ exactly yields (3.46)

$$\frac{1}{2\pi f} + \theta = \frac{n}{m}\,, \tag{4.18}$$

as expected, up to the sign which can be absorbed by the trivial redefinition of the variables. Since k and θ denote the noncommutativities of the world volume of D-instantons and the target space respectively, this result implies that they are closely related through the non-trivial equation (4.17), which can be naturally understood from the point of view of the twisted bundle on the noncommutative torus. Thus we confirm the proposal made in [6, 14, 18] that n D-instantons on the T^2 with B_{ij} flux can be described by the U(n) twisted gauge theory on the noncommutative torus by regarding the configuration of the D-instantons as that of a D-string in the context of the perturbative bosonic string theory. There m is interpreted as the winding number of the resulting D-string. The same interpretation is also made in [4, 13, 14, 18, 20] and seems natural for the following reason, for example. Setting $\theta = B_{12} = 0$ in (4.17) yields

$$2\pi k n = (2\pi\ell_s)^2 m \,, \tag{4.19}$$

which is consistent with the physical picture that each D-instanton has 'cell' of area $2\pi k$ because of the world volume noncommutativity (4.1) and the D-string wrapped m times over T^2 consists of n such cells.

5. Conclusions and discussions

We have constructed the boundary state which realizes the NCG representations of D-branes on noncommutative torus. From our construction the dual nature of the noncommutativity of the world volume and that of the target space can be seen naturally. Also our analysis is consistent with the conjecture that the gauge theories on the world volumes of D-branes compactified on a torus with the background 2-form field should be described by those on the noncommutative torus.

Some remarks are in order:

We have used a particular representation of the D-brane coordinates q^i

$$q^1 = -2\pi i\ell_s\partial_{\sigma_1} \,, \qquad q^2 = -2\pi i\ell_s(\partial_{\sigma_2} - if\sigma_1) \,. \tag{5.1}$$

There is an arbitrariness in the term proportional to f due to a gauge transformation by an arbitrary function of σ_j. It is an automorphism of the algebra on the noncommutative world volume because it preserves the algebra of q^i's and π_j's. Thus, in the sense of NCG, it should give (a part of) diffeomorphism of the world volume. Actually, a function of σ_j is nothing but a function of π_j so that it brings about a canonical transformation of q^i. This is also consistent with the discussions in [31] that the longitudinal translation of the D-brane corresponds to a kind of gauge transformation in the dual picture.

On the other hand, as noted below (3.20), one can modify the representation of the boundary state by a function of x^i which changes the expression of momentum conservation. Under the identification of the parameter θ with 2-form gauge potential, this transformation effectively gives gauge transformation of B_{ij}. From the string point of view, the gauge transformation of B_{ij} is related to the conformal transformation. In this sense, if noncommutative structures discussed here provide the space-time uncertainties [28, 34, 35] which are considered to reflect the conformal symmetries both in the world volume and the target space, then our result seems to be consistent with the proposal in [35] that two conformal symmetries play dual roles. It will be interesting to consider an interpretation of the relation (3.46) in the light of the space-time uncertainty.

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References

[1] E. Witten, *Nucl. Phys.* **B 460** (1996) 335 [hep-th/9510135].

[2] M.R. Douglas, D. Kabat, P. Pouliot and S.H. Shenker, *Nucl. Phys.* **B 485** (1997) 85 [hep-th/9608024].

[3] T. Banks, W. Fishler, S.H. Shenker and L. Susskind, *Phys. Rev.* **D 55** (1997) 5112 [hep-th/9610043];
For a review, see T. Banks, *Nucl. Phys.* **67** *(Proc. Suppl.)* (1998) 180 [hep-th/9710231];
O. Bigatti and L. Susskind, *Review of Matrix theory*, hep-th/9712072.

[4] W. Taylor IV, *Phys. Lett.* **B 394** (1997) 283 [hep-th/9611042];
O.J. Ganor, S. Ramgoolam and W. Taylor IV, *Nucl. Phys.* **B 492** (1997) 191 [hep-th/9611202].

[5] A. Connes, M.R. Douglas and A. Schwarz, *J. High Energy Phys.* **02** (1998) 003 [hep-th/9711162].

[6] M.R. Douglas and C. Hull, *J. High Energy Phys.* **02** (1998) 008 [hep-th/9711165].

[7] P.-M. Ho, Y.-Y. Wu and Y.-S. Wu, *Phys. Rev.* **D 58** (1998) 026006 [hep-th/9712201].

[8] P.-M. Ho and Y.-S. Wu, *Phys. Rev.* **D 58** (1998) 066003 [hep-th/9801147].

[9] R. Casalbuoni, *Phys. Lett.* **B 431** (1998) 69 [hep-th/9801170].

[10] M. Li, *Comments on supersymmetric Yang-Mills theory on a noncommutative torus*, hep-th/9802052.

[11] Y.-K.E. Cheung and M. Krogh, *Nucl. Phys.* **B 528** (1998) 185 [hep-th/9803031].

[12] T. Kawano and K. Okuyama, *Phys. Lett.* **B 433** (1998) 29 [hep-th/9803044].

[13] F. Ardalan, H. Arfaei and M.M. Sheikh-Jabbari, *Mixed branes and M(atrix) theory on noncommutative torus*, hep-th/9803067; *J. High Energy Phys.* **02** (1999) 016 [hep-th/9810072].

[14] P.-M. Ho, *Phys. Lett.* **B 434** (1998) 41 [hep-th/9803166].

[15] D. Bigatti, *Noncommutative geometry and super Yang-Mills theory*, hep-th/9804120.

[16] A. Schwarz, *Nucl. Phys.* **B 534** (1998) 720 [hep-th/9805034].

[17] G. Landi, F. Lizzi and R. J. Szabo, *String geometry and the noncommutatve torus*, hep-th/9806099.

[18] B. Morariu and B. Zumino, *Super Yang-Mills on the noncommutative torus*, hep-th/9807198.

[19] D. Brace, B. Morariu and B. Zumino, *Dualities of the Matrix model from T-duality of the type II string*, hep-th/9810099; *T-duality and Ramond-Ramond backgrounds in the Matrix model*, hep-th/9811213.

[20] C. Hofman and E. Verlinde, *J. High Energy Phys.* **12** (1998) 010 [hep-th/9810116].

[21] M.M. Sheikh-Jabbari, *Phys. Lett.* **B 450** (1999) 119 [hep-th/9810179]; *Open string in a B-field background as electric dipoles*, hep-th/9901080.

[22] D. Brace and B. Morariu, *J. High Energy Phys.* **02** (1999) 004 [hep-th/9810185].

[23] C. Hofman and E. Verlinde, *Gauge bundles and Born-Infeld on the noncommutative torus*, hep-th/9810219.

[24] A. Konechny and A. Schwarz, *BPS states on noncommutative tori and duality*, hep-th/9811159; *Supersymmetry algebra and BPS states of super Yang-Mills theories on noncommutative tori,* hep-th/9901077.

[25] C.-S. Chu and P.-M. Ho, *Noncommutative open string and D-brane*, hep-th/9812219.

[26] N. Kim, *Orientifolds of Matrix theory and noncommutative geometry*, hep-th/9901008.

[27] N. Ishibashi, H. Kawai, Y. Kitazawa and A. Tsuchiya, *Nucl. Phys.* **B 498** (1997) 467 [hep-th/9612115].

[28] T. Yoneya, *Prog. Theor. Phys.* **97** (1997) 949 [hep-th/9703078].

[29] A. Fayyazuddin, Y. Makeenko, P. Olesen, D. J. Smith and K. Zarembo, *Nucl. Phys.* **B 499** (1997) 159 [hep-th/9703038].

[30] S. Hirano and M. Kato, *Prog. Theor. Phys.* **98** (1997) 1371 [hep-th/9708039].

[31] N. Ishibashi, *Nucl. Phys.* **B 539** (1999) 107 [hep-th/9804163].

[32] K. Itoh, M. Kato, H. Kunitomo and M. Sakamoto, *Nucl. Phys.* **B 306** (1988) 362.

[33] M. Sakamoto, *Phys. Lett.* **B 231** (1989) 258.

[34] T. Yoneya, *Duality and Indeterminacy Principle in String Theory* in *Wandering in the Fields*, eds. K. Kawarabayashi and A. Ukawa, World Scientific, 1987, p. 419; *String Theory and Quantum Gravity* in *Quantum String Theory*, eds. N. Kawamoto and T. Kugo, Springer, 1988, p.23; *Mod. Phys. Lett.* **A 4** (1989) 1587.

[35] A. Jevicki and T. Yoneya, *Nucl. Phys.* **B 535** (1998) 335 [hep-th/9805069].